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Aircraft Elastic Mode Control

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Introduction

IN 1914, a patent was granted for a "Stabilizing Device for Flying Machines" which would "counteract the disturbance (gust) and prevent it from having an injurious effect on the stability of the machine."[1] Since then over thirty other U.S. patents related to gust load alleviation have been issued. During the same time period, advances in aerodynamic, structural, propulsion, and control systems technology have resulted in two orders of magnitude increase in speed and altitude regimes of aircraft flight. This performance increase has not been matched by improved aircraft gust response characteristics. Since atmospheric turbulence cannot be avoided entirely, airplanes have been constructed to withstand gust loads. The aircraft dynamic response resulting from gust inputs is usually considered only for determining the estimated additional structural strength needed for survival. At the present time about one structural failure in 10^8 flying hours is expected and accepted. As yet, the benefits to be gained from gust alleviation control systems have not been realized since such systems have not been applied to a production airplane.

A spectacular illustration of the effects of gusts on aircraft occurred on January 10, 1964. A B-52H flying at low altitude encountered a patch of severe turbulence. During the first 5.7 sec, its rigid-body and elastic mode dynamic responses built up until, under a combination of rigid-body and elastic mode excursions, the tail loads exceeded ultimate design values. This B-52 lost 85% of its vertical tail because of the gusts with an estimated peak velocity of 120 fps. Under these circumstances its yaw damper was saturated, resulting in virtually an unaugmented rigid-body dynamic response. The gusts did not fail the tail, but they excited responses that did. Amazingly, the pilot was able to land this airplane.

The effects of turbulence on large flexible aircraft have led to concern in four major areas: ride qualities (crew and passenger comfort as a function of vibration level), structural fatigue life, peak structural loads, and handling qualities. There is also the additional concern of aircraft upsets which on occasion has resulted in disastrous vehicle-pilot-control system interaction.[2,3] With the advent of highly flexible airplanes, such as the B-52, XB-70, C-5A, and SST, there is a need for more than rigid-body gust alleviation. Turbulence feeds as much or more energy into the lower-frequency normal elastic modes of these type airplanes as it does into the rigid-body dynamics. Thus, there is a demonstrated need for elastic mode control systems which can suppress the level of normal accelerations at selected locations on the structure. This paper will attempt to survey the research and development which has been done on such systems. It is limited to results in the open literature; although, significant work has been done and documented in several classified reports.

Rigid-body gust alleviation research over the years has been extensive and varied, notwithstanding the fact that no system has been incorporated into a production airplane. In 1938, Rene Hirsch worked out a system which was test flown much later in 1954.[4] In 1950, Douglas Aircraft Company was flying a C-47 equipped with gust alleviation flaps.[5] In 1954, Cornell Aeronautical Laboratory flew a lateral-directional system in a PT-26.[6] NACA performed a significant series of demonstration flights in a C-45 starting in 1952.[7,8] In England, a Lancaster bomber was flown with a flap gust

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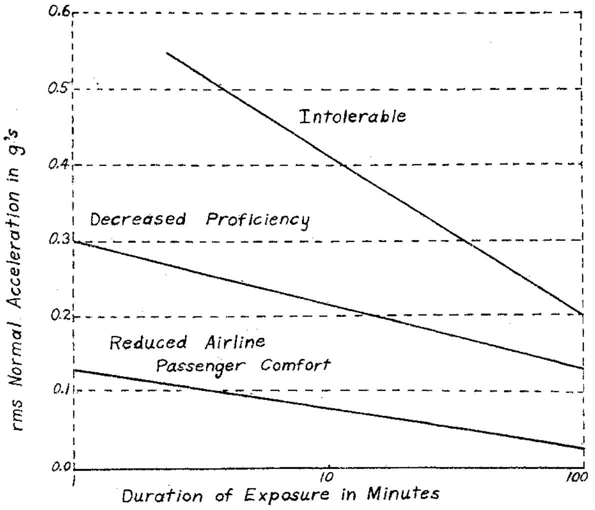

Fig. 1 Time-intensity vibration boundaries.

alleviation system before 1957.[9,10] These typical flight results were augmented by many more system analyses and some wind-tunnel tests. Reference 11 is an excellent bibliography on gust alleviation and low-altitude turbulence and contains 263 references with abstracts.

A rigid-body gust alleviation system and an elastic mode control system are both only part of the over-all vehicle flight control system, which also includes the rigid-body stability augmentation system needed for satisfactory rigid-body handling qualities and the usual autopilot components. However, structural flexibility can affect these other aspects of flight control design. Static aeroelastic deflections of the structure modify aerodynamic pressure distributions over the aircraft, which implies changes to the stability derivatives associated with the rigid body, small perturbation, dynamic equations of motion used in stability augmentation system design. The early attempts to account for aeroelastic effects on aircraft stability and control took the approach of making static aeroelastic corrections to the stability derivatives. References 12–15 are representative of this work.

When the dynamic effects of flexibility are sufficiently important that they must be included as additional degrees of freedom, a common approach has been to approximate the dynamics by a truncated set of superimposed orthogonal normal vibration modes. In this case, the phenomena of most interest are the effects of aerodynamic coupling between the various elastic modes and between elastic and rigid-body modes and also the elastic mode interactions with the feedback control system. Reference 16 was one of the earliest comprehensive studies of this problem. Later useful work was documented in Refs. 17–23.

Ride Qualities

Airplane riding quality or passenger comfort depends on many interrelated factors but primarily on motion. The most important motion parameters are vibration frequencies and duration of exposure to various acceleration levels. There are a number of secondary factors which also influence comfort, such as cabin ventilation and pressurization, psychological or mental attitudes, and crew or passenger work loads. It is known that military pilots will change altitude and/or reduce speed when rms normal acceleration levels exceed about 0.5 g for several minutes.[24] At this level, they suffer considerable discomfort, reduced piloting efficiency, and concern for aircraft structural integrity. For levels over 0.2 g, such tasks as instrument reading and writing are difficult. In general, human tolerance to lateral acceleration intensities is about one-half that of vertical accelerations. However, such factors as seat restraints, head rests, and frequency content of the acceleration environments will alter this ratio.

Figure 1[24] and Fig. 2[25] present curves that summarize data on human tolerance of vertical vibration. The diagonal scales of Fig. 2 are for various levels of rms vertical displacement, and the shaded areas are indicative of a measure of uncertainty on the exact location of each boundary. As indicated in these figures, ride quality is measured in terms of the rms normal acceleration level or intensity occurring at various crew and/or passenger stations in the airplane. The random acceleration experienced at these stations in atmospheric turbulence is a result of the transfer of energy from the air to the aircraft, inducing rigid-body and elastic mode dynamic responses.

Data on aircraft such as the XB-70 and the proposed SST indicate that the lower-frequency symmetric elastic modes contribute significantly to the acceleration levels at the crew station and certain passenger stations along the fuselage. For example, in one SST configuration it was estimated that about 60% was due to the first three elastic modes, 30% because of the rigid-body response, and 10% because of the higher-frequency elastic modes at a particular fuselage station.[26] An elastic mode control system designed to suppress acceleration levels at selected stations would provide a significant improvement in ride quality. Additional information on the effects of motion on human comfort and ride quality is given in Refs. 27–31.

Handling Qualities

The subject of handling qualities requirements and criteria for highly elastic aircraft in turbulent and high dynamic pressure environments has been largely ignored by researchers. Much of the research on handling qualities has been concerned with relatively rigid, tactical military aircraft. The handling qualities parameters, such as short-period and dutch-roll frequencies and damping ratios, ϕ/β and ω_ϕ/ω_d ratios, which have been determined pertinent for such airplanes, can become largely meaningless for a flexible airplane with elastic frequencies close to the so-called rigid-body frequencies. When multiple frequencies are in close proximity, the pilot cannot easily discern individual modes of motion; rather his liking or disliking for the transient dynamics will likely be based on the time histories of the total motion. No performance criteria suitable for handling qualities specification are presently available for such higher-order responses. This is all too evident in that no useful discussion of aeroelas-

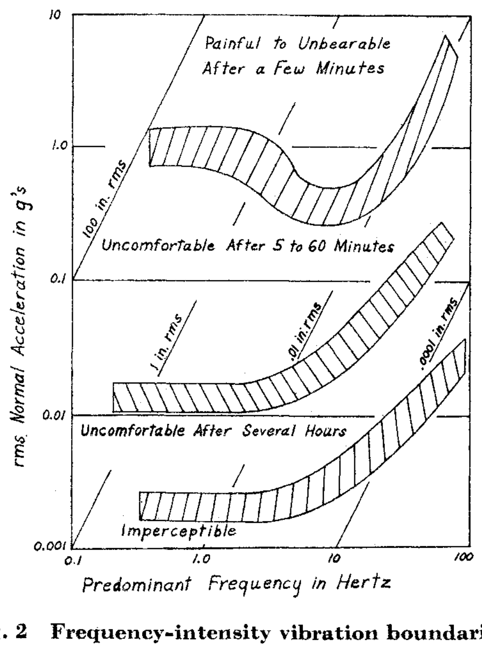

Fig. 2 Frequency-intensity vibration boundaries.

tic effects is included in the 1969 revision to the military aircraft handling qualities specification.[32]

A question which cannot now be answered, but one worthy of some research, is how do elastic mode frequencies, amplitudes, and damping ratios influence pilot rating of handling qualities? It would appear to be fundamental, and possibly even critical to successful design of such highly elastic aircraft as the Boeing SST and the Air Force B-1A.

Reference 33 documents results of simulation and flight test of various B-52 lateral-directional stability augmentation systems on rigid-body handling qualities in turbulence. However, no mention is made of what influence the elastic modes of the B-52 may have had on the rigid-body pilot opinion ratings. What constitutes good dutch-roll frequency and damping ratio in various turbulence intensities might very likely be different for a rigid airplane than for an elastic one.

Power Spectral Density Analysis

Turbulence being a random process is most suitably modeled mathematically using generalized harmonic analysis or power spectrum methods, and the output variable responses are generally determined as rms values.[34,35,36] When turbulence hits a flexible airplane, both the rigid-body and elastic modes are excited. Figure 3 shows the problem schematically.[37] The energy in the turbulent atmosphere is represented by the typical power spectral density curve of gust velocity vs gust wave number. Only the vertical component w_g is shown; similar curves are available for the lateral and longitudinal components of gust velocity. The curve below shows the relationship of the rigid-body short-period frequency relative to the lower elastic mode frequencies for a typical modern flexible airplane configuration. The ordinate of the curve is the magnitude of the normal acceleration to gust velocity frequency response transfer function for a selected fuselage station. At a given speed, a frequency can be related to the gust power spectrum through the gust wave number. The figure shows that gust energy is available to excite the rigid-body motion as well as the elastic modes. The rms value of normal acceleration is then given by

$$\sigma_{n_z} = \left[\int_0^\infty |n_z/w_g|^2 \Phi_{w_g}(\omega)d\omega\right]^{1/2} \tag{1}$$

Structural Loads and Fatigue

When ultimate loads and stresses are needed, a useful formulation using PSD methods is the number of exceedances per unit time $N(y)$ of a specific value of a parameter y.[37]

$$N(y) = N_0\{P_1 \exp[-y/(\sigma_y/\sigma_{w_g})b_1] + P_2 \exp[-y/(\sigma_y/\sigma_{w_g})b_2]\} \tag{2}$$

where y = response parameter value.

N_0 = characteristic frequency =

$$U_0/2\pi \left[\int_0^\infty \Phi_y(\Omega)\Omega^2 d\Omega \Big/ \int_0^\infty \Phi_y(\Omega)d\Omega\right]^{1/2} \text{ in Hz}$$

$$\Phi_y(\Omega) = \left|\frac{y}{w_g}\right|^2 \Phi_{w_g}(\Omega); \quad \sigma_{w_g} = \left[\int_0^\infty \Phi_{w_g}(\Omega)d\Omega\right]^{1/2}; \quad \sigma_y = \left[\int_0^\infty \Phi_y(\Omega)d\Omega\right]^{1/2}$$

P_1,P_2 = proportion of total time spent in disturbed environment; 1) normal environment, 2) severe environment and b_1,b_2 = scale parameter in probability density distribution of rms of environmental velocity. Either σ_y or $N(y)$ can be used in loads evaluation. For fatigue evaluation, Minor's hypothesis is often used. Detailed stress characteristics in the form of number of cycles to failure at a given stress level for a given material ($S - N$ curves) are required. Failure is hypothesized to occur when

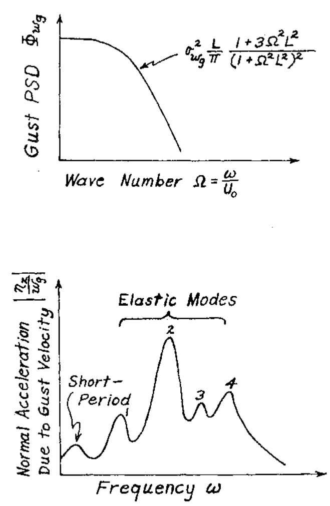

Fig. 3 Turbulence power spectral density and airplane dynamic response.

$$\sum_{i=1}^{n} \frac{N(y)_i}{\bar{N}(y)_i} = 1 \tag{3}$$

where y = stress level, $N(y)_i$ = applied number of cycles at stress level i, $\bar{N}(y)_i$ = allowable number of cycles at stress level i, and n = number of spectrum levels.

Elastic Mode Control

Given an airplane configuration, there are three basic ways of altering the gust energy acting on the airplane. 1) Reduction of gust sensitivity can be accomplished by use of rate and acceleration sensors to generate feedback control signals which drive lift controls that cancel the gust induced lift increment. Direct lift control (DLC) holds considerable promise for such rigid-body gust alleviation systems. 2) A second method for preventing gust energy being absorbed is to increase the resonant frequencies out of the high-energy, low-frequency portion of the gust velocity spectrum by artificial stiffening with feedback controls. Increasing the closed-loop short-period frequency reduces normal acceleration response forward of the center of gravity and behind the rigid-body node because the airplane can weather-cock more quickly into the wind, reducing the angle of attack. The same principle applies to alleviating elastic mode response. 3) The third and probably most effective method is to damp the gust excited elastic modes and dissipate the gust energy. Artificial damping is the most common form of stability augmentation for rigid-body control.

For purposes of improving ride quality, augmentation of elastic mode damping will result in lower rms normal acceleration levels. However, the amplitude and frequency of the elastic mode response may still be objectionable from a peak structural loads and handling qualities viewpoint. Therefore, frequency augmentation may also be necessary. This can be illustrated by the following simple analysis.

A representative general form for the transfer function of an elastic mode generalized displacement $\eta(t)$ to gust induced angle of attack α_g is

$$\eta(s)/\alpha_g(s) = K/(s^2 + 2\zeta\omega s + \omega^2) \tag{4}$$

where K is a gain constant and ω and ζ are the elastic mode coupled undamped natural frequency and damping ratio. For simplicity, assume α_g is a unit impulse; then it can be shown that

$$\eta(t) = [-Ke^{-\zeta\omega t}/\omega(1 - \zeta^2)^{1/2}] \times \cos[\omega(1 - \zeta^2)^{1/2}t + (\pi/2)] \tag{5}$$

Equation (5) gives the transient time history of $\eta(t)$ because of a unit impulse gust input. In order to determine the peak

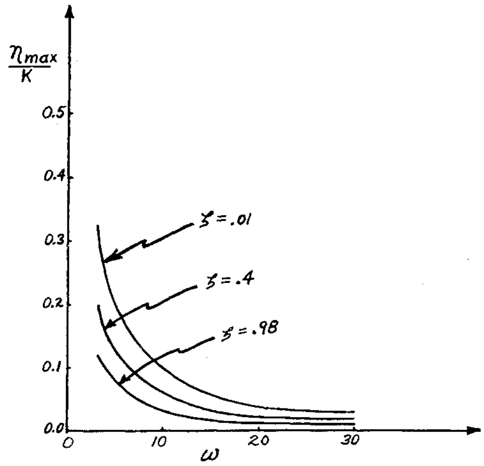

Fig. 4 Plot of Eq. (7).

instantaneous values of $\eta(t)$ as a function of ω, ζ, and K, set the time derivative of Eq. (5) equal to zero. The peak value occurs at

$$t = \{\arctan[-\zeta/(1 - \zeta^2)^{1/2}] - (\pi/2)\}/\omega(1 - \zeta^2)^{1/2} \quad (6)$$

Substituting Eq. (6) into Eq. (5) gives

$$\underset{\max}{\eta(t)} = \frac{-K}{\omega(1 - \zeta^2)^{1/2}} \exp\left[\frac{-\zeta}{(1 - \zeta^2)^{1/2}} \times \left(\arctan\frac{-\zeta}{(1 - \zeta^2)^{1/2}} - \frac{\pi}{2}\right)\right] \cos\left(\arctan\frac{-\zeta}{(1 - \zeta^2)^{1/2}}\right) \quad (7)$$

Equation (7) is plotted in Fig. 4. One sees that for elastic modes with coupled frequencies below about 8 rad/sec, the peak response can be cut down more by frequency augmentation than by damping augmentation, all other design considerations being equal—which of course they will not be.

Design and synthesis of an aircraft flight control system which includes subsystems to control structural dynamic response and provide rigid-body gust alleviation is in general an extremely complex multi-input, multi-loop, multivariable, analysis problem and involves additional complications such as accounting for unsteady aerodynamics effects and selection of suitable input forces and sensor locations. The elastic mode and rigid-body gust alleviation portions of the total system must be compatible with the rigid-body SAS and not interfere with pilot inputs for maneuvers. A rough idea of the degree of complexity involved in synthesizing such systems can be gained by considering the block diagram of Fig. 5. The outputs are rigid-body pitch response θ and two elastic mode generalized displacements η_1 and η_2. Two input forces, represented by signals δ_1 and δ_2, are provided for controlling the response, δ_1 for rigid motion and δ_2 for elastic motion. The transfer functions G_1, G_2, and G_3 include all required sensing, filtering, shaping, and actuation dynamics. θ_c is some desired pitch command input. Any actual system of this character could be manipulated into the unity feedback form shown.

The θ/θ_c closed-loop transfer function can be written as

$$\theta/\theta_c = G_4/(1 + G_4) \quad (8)$$

where

$$G_4 = G_1 \left\{\left(\frac{\theta}{\delta_1}\right) + G_2\left[\left(\frac{\theta}{\delta_1}\right)\left(\frac{\eta_1}{\delta_2}\right) - \left(\frac{\theta}{\delta_2}\right)\left(\frac{\eta_1}{\delta_1}\right)\right] + G_3\left[\left(\frac{\theta}{\delta_1}\right) \times \left(\frac{\eta_2}{\delta_2}\right) - \left(\frac{\theta}{\delta_2}\right)\left(\frac{\eta_2}{\delta_1}\right)\right]\right\} \Big/ \left[1 + G_2\left(\frac{\eta_1}{\delta_2}\right) + G_3\left(\frac{\eta_2}{\delta_2}\right)\right] \quad (9)$$

The ratios in parentheses indicate transfer functions. Equation (8) can be realized by two successive loop closures shown in Fig. 6.

The open-loop poles and zeros of Closure 1 will be complicated functions of elastic mode sensor and actuator locations on the aircraft. Root locus analyses can be used on Closure 1 to determine sensor and actuator locations which yield the desired elastic mode characteristics in terms of damping ratio and frequency augmentation. The closed-loop roots of Closure 1 determine the open-loop poles and zeros of Closure 2, which is then made as an outer loop to yield the desired rigid-body pitch response. Iterations in these two closure procedures will likely be required to arrive at the final system.

With this relatively simple example, one can see that synthesis of elastic mode control systems using conventional servo analysis techniques is a very difficult procedure. The particular procedure outlined previously is by no means the only one available which uses conventional servo analysis techniques; however, it is representative of the complexity of all such approaches. As will be discussed later, linear optimal control theory holds much promise as a better design tool.

One of the system interface problems to be solved is that between the regular stability augmentation system (SAS), needed by most modern aircraft for satisfactory handling qualities, and the elastic mode control system. To produce the most efficient system, the SAS design, ride quality, and fatigue life improvements should be considered on an integrated design basis. However, failure of the elastic mode control system cannot be allowed to degrade SAS performance below acceptable safety limits. Thus, it is desirable that these systems operate independently. To accomplish such independence will usually mean that the sensor outputs must be separated into rigid and elastic components. There are several ways of accomplishing this, but basically they fall into two categories, 1) filtering of the sensor outputs[38,39] and 2) blending of multiple sensor outputs.[40]

The necessary control forces are usually generated with aerodynamic control surfaces. Theoretically, the most effective location for the force is at the point of maximum elastic mode displacement. The locations must be selected to effectively control the response at the desired stations, however. The forces required are not large. Conventional surfaces usually are sized and have deflection limits set by low-speed maneuver requirements to far larger values than needed for elastic mode control. However, the deflection rates required can become a problem since the surface must move with a frequency at least as high as the highest mode for which control is desired. Generally, though, state-of-the-art surface actuators will be adequate since most of the modal frequencies of interest are below 5 Hz.

Review of Past Programs

Some of the earliest analysis work on elastic mode control systems was done by Autonetics Division of North American Aviation Inc.[41,42] They referred to their technique as "dynamic rigidity augmentation." It was based on the use of servo control systems and existing aircraft control surfaces

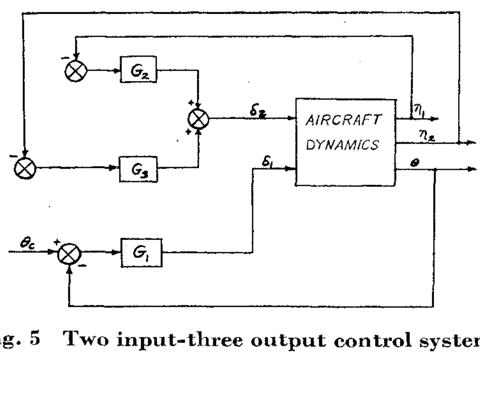

Fig. 5 Two input-three output control system.

to augment elastic mode damping and frequency. The work demonstrated the conceptual feasibility of such control techniques and sparked a great deal of the later interest and research programs sponsored by the Air Force Flight Dynamics Laboratory and the aircraft industry. However, by not considering such factors as unsteady aerodynamics, varying flight conditions, and force input and sensor coupling effects, practical feasibility of such control systems remained to be determined in later work.

The practical feasibility of elastic mode control for advanced aircraft was established by North American Aviation/Los Angeles Division in a program completed in 1965.[43] The XB-70 was used as the study vehicle, and the program considered such important practical problems as: 1) actual flight profiles and the resulting Mach number and altitude effects, 2) means of applying the required energy to the structure to provide effective damping (i.e., what control surfaces or devices does one use to generate the generalized forces?), 3) mass and mass distribution changes during flight which influence elastic mode undamped natural frequencies, 4) lag effects of sensors, servos, actuators, and aerodynamics, 5) power requirements of the elastic mode control system. They developed a technique for adding damping to the lower-frequency elastic modes based on the concept of using a generalized dissipative force at the same location where the sensor measures elastic mode acceleration. It is termed "Identical Location of Accelerometer and Force (ILAF)." The technique makes use of modal coupling effects rather than attempting to eliminate coupling and results in a relatively simple, conventional type control system which continuously guarantees phase and gain stabilization of the elastic modes, even with augmentation system failures.

In 1968–69, an ILAF elastic mode control system, designed by North American Rockwell, was installed and flight tested in the XB-70 under a jointly sponsored NASA/Air Force program.[37] The elevons were used to control the symmetric elastic modes, with the objective of demonstrating improved ride quality at several fuselage stations through reduction of rms normal accelerations. The author is aware of no published results on the success of this experimental program; private conversations with North American Rockwell engineers revealed successful early test results which verified design predictions. Unfortunately, the test program was terminated before completion due to retirement of the XB-70 to the Air Force Museum at Wright-Patterson Air Force Base.

In a follow-on effort sponsored by the Air Force, North American Rockwell Corp. completed in 1967 the Gust Alleviation and Structural Dynamic Stability Augmentation System (GASDSAS) analytical design program. The objective was the preliminary design of a control system which was capable of actively reducing the symmetric and antisymmetric normal accelerations due to both rigid-body (gust alleviation) and elastic modes of a low load factor, flexible aircraft flying at high speeds in turbulence, such as the proposed B-1A (AMSA) airplane. In addition, the compatibility of such a control system with terrain-following, handling qualities, and pilot display requirements was evaluated using an analog simulator. Force generators used were elevator, rudder, ailerons, and flaps. In addition, small horizontal and vertical canards were shown to be extremely effective for elastic mode control. The system performance showed significant reductions in load factors at the pilot station and other fuselage stations, reduced wing loads, and favorable effects on flutter characteristics. The details of the over-all program results are contained in several Air Force Flight Dynamics Laboratory classified reports; however, there are two unclassified references available.[44,45]

Honeywell Inc. carried out an analytical program aimed at design of an elastic mode control system (using four symmetric modes) for the proposed Lockheed SST with the goal of ride quality improvement.[26] Their technique specified a set of requirements for the sensor configuration and filtering for corrective control using the elevons. Vehicle data were required only in a frequency response format, thus, facilitating the inclusion of unsteady aerodynamics. The system achieved the objective of a 50% reduction in rms normal acceleration at the critical fuselage stations.

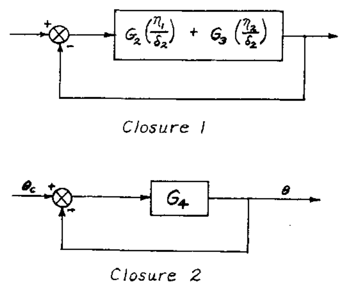

Fig. 6 The θ/θ_c by loop closures.

An Air Force sponsored program was conducted by The Boeing Company from 1964 to 1967 to develop a B-52 flight control and stability augmentation system that would provide improvements in structural fatigue life and in rigid body and elastic mode stability in severe turbulence.[33,46–48] A prototype of the advanced pitch and yaw SAS was installed on a B-52H and flight test results showed significant reductions in dynamic response to turbulence. Damping of the rigid-body modes and low-frequency antisymmetric aft fuselage elastic modes was increased. Lateral loads on the fin and aft fuselage were reduced more than 20% in turbulence. Fatigue damage rates were reduced more than 50% for these same locations. Test results agreed well with theoretical predictions. This system is now available in kit form and is being retrofitted into the B-52G and H fleet.

Still another program involving a B-52 is the Loads Alleviation and Mode Stabilization (LAMS) program conducted by The Boeing Company in conjunction with Honeywell Inc. under the sponsorship of the Air Force Flight Dynamics Laboratory.[49,58] Improvement in fatigue damage rate and load reductions were the major performance goals. The total system included both longitudinal and lateral-directional SAS and elastic mode control. Existing control surfaces were used, including wing spoilers, but actuation modifications were made to allow higher-frequency response and symmetric deflection of ailerons. Flight test results indicated major reductions in fatigue damage rates and wing structural loads. The goal of 50% reduction in fatigue damage rates was achieved for all sensitive aircraft structure.

Linear Optimal Control

Probably the first application of optimal control theory to gust alleviation was in 1957.[50] Use was made of Wiener optimum filter theory to synthesize a control system that would minimize the center of gravity rms normal acceleration and rms pitching velocity. Linear optimal control theory holds considerable promise for eliminating much of the trial and error of servo analysis.[45,51–56] In a sense, this theory is the reverse of conventional servo design procedure in that it allows the selection of desired closed-loop poles based on minimization of a quadratic performance index and then solves for the feedback control law which yields this closed-loop response. For a completely observable (measurable) and controllable plant state, a single control input force can theoretically achieve any desired closed-loop characteristic equation in terms of frequencies and damping ratios and real roots of all the modes of motion included. However, for a plant as complicated as a flexible aircraft, it is unlikely that the resulting control law will be physically realizable; a suboptimal approximation to the control law will be usually necessary and frequently desirable. In order to provide an awareness of what is involved, one of the more promising methods is briefly outlined below.[51]

The objective of the theory is to minimize a quadratic performance index of the form

$$P.I. = \int_0^\infty (y^T Q y + u^T R u) dt \tag{10}$$

which includes weighted measures of the output variables y (such as pitch angle θ and elastic mode generalized displacement η) and the control input variables u. The superscript T in Eq. (10) indicates the transpose of column vectors y and u. The flexible aircraft equations of motion can be reduced to a set of first-order differential equations in the state variables x.

$$x = Fx + Gu \tag{11}$$

F and G are constant matrices. Various optimization techniques can be used to determine u which minimizes $P.I.$ The weighting matrices Q and R are chosen, through use of root square locus plots to yield desired closed-loop roots.[53] For given Q and R, the feedback control law will be a unique linear combination of the states x and guarantees a stable system which has the desired response.

It is likely that the control law will not be directly realizable. Practical sensors will necessarily measure combinations of the states rather than each state individually. It is therefore necessary to express the control law in terms of measurable quantities from gyros and accelerometers. These quantities y can then be related to the states x through a known linear transformation H. The optimum control law is then

$$u_0 = -Lx = -LH^{-1}y \tag{12}$$

where matrix L contains the feedback gains given by the optimization, which can be carried out either in the time domain involving solution of a matrix Riccati equation[53,56] or in the frequency domain involving solution of a matrix Wiener-Hopf equation.[45,54]

Concluding Remarks

Control systems for alleviating an airplane's rigid-body and elastic mode responses to turbulence have been proved feasible by comprehensive analysis and extensive flight test. Substantial improvements in riding qualities, structural fatigue life, and peak loads have been demonstrated. Such systems are extremely sensitive to parameter variations, however, and must be carefully designed to prevent degraded responses and instabilities under offdesign conditions.

The designer has several alternative approaches to achieve his design goals, involving a variety of components and system concepts. However, none of these approaches change the complex multiloop, multivariable design problem. Closely coupled loops require detailed attention and the fullest use of modern computer equipment. Improvements in design procedures in the form of better performance criteria, more accurate aerodynamic analytical techniques, and optimal control design are expected to improve design confidence and lower costs to acceptable levels. The author gratefully acknowledges the many sources of information listed in the references on which this paper is based. Particularly helpful was the material of Refs. 37 and 57.

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